Subscripts

0 = initial condition
1 = aqueous phase
2 = organic phase

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Estimation of Bubble Diameter in Gaseous Fluidized Beds

Bubble size is one of the most important parameters in the design and simulation of a fluidized-bed reactor.

A correlation of the bubble size and growth in fluidized beds of various diameters is developed. A maximum bubble diameter determined from the bubble coalescence is incorporated in the correlation to relate the effect of the bed diameter on the bubble size.

Experimental data of bubble size reported are used to develop and test the validity of the correlation. The bubble diameters calculated using this correlation show good agreement with the observed bubble diameters.

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SCOPE

A correlation of bubble diameter for fluidized beds of various sizes including pilot scale is presented.

In recent years a number of fluidized-bed models (Mori and Muchi, 1972; Kato and Wen, 1969; Kunii and Levenspiel, 1968; Toor and Calderbank, 1967; Partridge and Rowe, 1966; Kobayashi and Arai, 1965; Orcutt et al., 1962) which take into consideration the behavior of bubbles have been proposed for predicting the performance of fluidized-bed reactors. Ishida and Wen (1973), Wen (1972), and Yoshida and Wen (1970) have also pointed out the importance of the bubble behavior in the coal conversion processes such as gasification and combustion.

In these studies one of the most important factors governing the extent of chemical conversion is the diameter of bubbles in the bed.

Although many correlations for estimation of the bubble diameter in fluidized beds (Yasui et al., 1958; Kato and Wen, 1969; Park et al., 1969; Whitehead et al., 1967; Rowe et al., 1972; Geldart, 1971; Chiba et al., 1973) are available, none of these correlations can predict the effect of the bed diameter on the bubble diameter.

In this paper, the bubble size and bubble growth rate are examined in light of the bed diameter and the design of distributor plates. A semi-empirical equation for bubble growth in fluidized beds of various sizes including pilot-plant scale is presented.

CONCLUSIONS AND SIGNIFICANCE

A correlation which predicts bubble diameters in freely bubbling fluidized beds and which accounts for the effect of the bed diameter on the bubble diameter is presented.

The proposed bubble growth correlation has the form

$$\frac{D_{BM} - D_B}{D_{BM} - D_{B0}} = \exp(-0.3h/D_t)$$

where D_B is the diameter of the bubble, D_t is the bed diameter, and h is the elevation or the height above the distributor plate. Initial bubble diameter formed at the surface of the perforated plate is calculated from

$$D_{B0} = 0.347\{A_t(u_0 - u_{mf})/n_d\}^{2/5}$$

where A_t is the cross-sectional area of the bed, u_0 is the

superficial gas velocity, u_{mf} is the minimum fluidization velocity, and n_d is the total number of orifices on the plate.

The value of D_{B0} for porous plate distributor can be evaluated from

$$D_{B0} = 0.00376(u_0 - u_{mf})^2$$

The diameter D_{BM} is the bubble diameter that would exist in a fluidized bed if all the fluidizing gas above that required for minimum fluidization went to form a single train of bubbles rising along the center line of the bed. D_{Bm} can be calculated from

$$D_{BM} = 0.652\{A_t(u_0 - u_{mf})\}^{2/5}$$

As shown in Figure 5, the correlation predicts experimental bubble diameter obtained by various investigators fairly accurately, especially for the bed with a diameter $30 < D_t < 130$ cm over the following variable ranges:

$$0.5 < u_{mf} < 20 \text{ cm/s}$$
$$0.006 < d_p < 0.045 \text{ cm}$$
$$u_0 - u_{mf} \quad 48 \text{ cm/s}$$

where d_p is the particle diameter.

Unlike previous bubble diameter correlations, this correlation incorporates the effect of bed diameter and design of perforated plate distributors on the bubble diameter. This correlation may be used to analyze the data from pilot-plant tests and can be used for design of a pilot-scale fluidized-bed reactor.

PREVIOUS WORK

Various correlations for estimating bubble diameters in fluidized beds have appeared in the literature and are summarized in Table 1. Most of these correlations are derived from data obtained from relatively small diameter beds. Therefore, these correlations are not useful in predicting the change in the bubble diameter when the bed diameter is changed.

It has been observed that the diameter of the bed does have a significant effect on the bubble diameter. For example, equivalent bubble diameters calculated from the bubble volume data reported by Werther (1973) for two widely differing bed diameters as shown in Figure 1 indicate that the smaller diameter bed consistently gives significantly larger bubbles at a given height than that in the larger diameter bed. In both of these cases the distributor geometry and the superficial gas and minimum fluidization velocities were the same. This observation has obvious practical implications in the scale-up of fluidized-bed reactors; therefore, correlations of bubble size should reflect this characteristic of bubbles in fluidized beds.

Theoretical analysis of the growth and coalescence of bubbles in fluidized beds by previous investigators (Chiba et al., 1973; Clift and Grace, 1972; Miwa et al., 1971) has indicated that the bubble diameter is a function of the bed diameter D_t, the distance of the bubble above the distributor h, and the initial bubble diameter D_{B0}. In the following sections the relationship between the bubble diameter D_B and these variables will be examined. A relationship involving the quantity D_{BM}, the maximum attainable bubble diameter which can be obtained by total coalescence of bubbles in the bed, is developed. This quantity and its relationship to the bubble diameter will be described in detail.

TABLE 1. SUMMARY OF CORRELATIONS FOR BUBBLE DIAMETER IN FLUIDIZED BEDS

Yasui et al. (1958)	$D_B = 1.6\rho_P d_P \left(\frac{u_o}{u_{mf}} - 1\right)^{0.63} \cdot h$
Kato and Wen (1969)	$D_B = 1.4\rho_P d_P \left(\frac{u_o}{u_{mf}}\right) h + D'_{Bo}$
Park et al. (1969)	$D_B = 33.3\, d_P^{1.5} \left(\frac{u_o}{u_{mf}} - 1\right)^{0.77} h$
Whitehead et al. (1967)	$D_B = 9.76 \left(\frac{u_o}{u_{mf}}\right)^{0.33(0.032h)^{0.54}}$
Rowe et al. (1972)	$D_B = -A + Bh + C\left(\frac{u_o}{u_{mf}}\right) + Dh\left(\frac{u_o}{u_{mf}}\right) + E\left(\frac{u_o}{u_{mf}}\right)^2$
Geldart (1971)	$D_B = D'_{Bo} + 0.027\,(u_o - u_{mf})^{0.94}\, h$
Chiba et al. (1973)	$D_B = D_{Bo}''\{2^{7/6} - 1)\,(h - hB_o)/D_{Bo}'' + 1\}^{2/7}$ for $h < h_k$*

* Numerical method is used to calculate D_B for $h > h_k$
$D_{Bo}' = (6G/\pi)^{0.4}/g^{0.2}$ and $D_{Bo}'' = (6G/\pi k_B)^{0.4}/g^{0.2}$
where A, B, C, D, E and k_B are constants determined by the properties of the solid particles; h_{Bo} is the height of the jet above the distributor, (cm); and h_k is the height from the bottom of the bed where the bubble radius becomes equal to the pitch of the holes in the distributor, (cm).

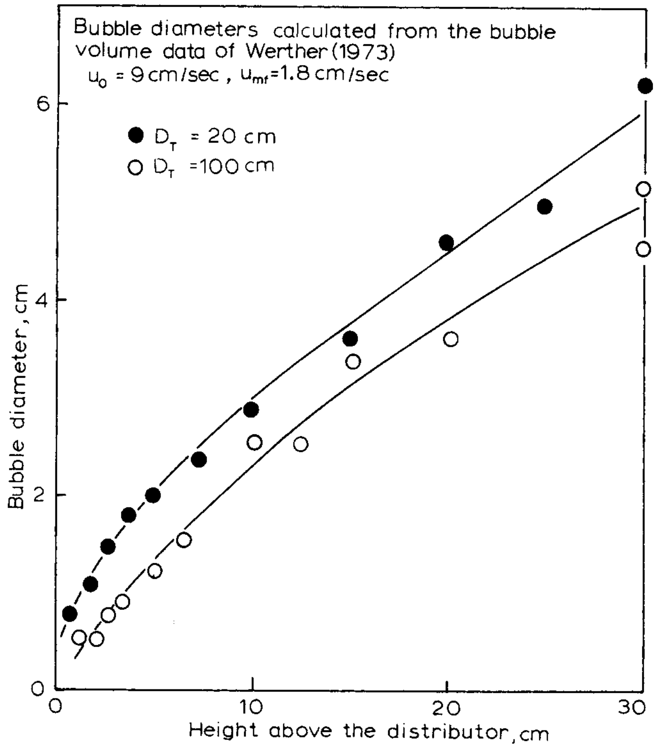

Fig. 1. Comparison of bubble diameters for two fluidized beds having different bed diameters by similar gas flow rates and geometrically similar distributor plates.

INITIAL BUBBLE DIAMETER, D_{B0}

The initial bubble size has obvious importance in determining bubble diameter within the fluidized bed since it is from this base size that a bubble will grow as it rises from the distributor plate up through the fluidized bed. Several correlations have been proposed for the initial bubble size from various types of distributors. The most important of these are probably the correlations of Miwa et al. (1971) for both perforated and porous plates.

Miwa et al. (1971) have extended Davidson and Schuler's (1960) theoretical development of bubble formation from a single nozzle to the formation of bubbles at the surface of a perforated plate. Miwa's correlation for the initial bubble diameter for a perforated plate is given by

$$D_{B0} = 0.347 \{A_t(u_0 - u_{mf})/n_d\}^{2/5} \tag{1}$$

Several other investigators (Basov et al., 1962; Chiba et al., 1973; Cooke et al., 1968) have also developed correlations for the initial bubble diameter for a perforated plate. These correlations agree in most cases closely with the correlation of Miwa et al. [Equation (1)].

Miwa et al. (1971) also developed an equation for the bubble size formed at a porous plate distributor:

$$D_{B0} = 0.00376 (u_0 - u_{mf})^2 \tag{2}$$

A comparison of the calculated initial bubble diameter D_{B0} with the experimentally observed bubble size just above the distributor plate is shown in Figure 2. It can be seen from this figure that the experimentally observed initial bubble diameters can be reasonably represented by Equation (1). However, as can be seen from Figure 2, the initial bubble diameter D_{B0} for a perforated distributor plate appears to become greater than that calculated from Equation (1) if D_{B0} calculated using Equation (1) is less than that computed from Equation (2). The observed values of D_{B0} under this condition appear to follow the line given by Equation (2) which is the initial bubble diameter correlation for a porous plate. More data are needed to substantiate the relationship shown in Figure 2.

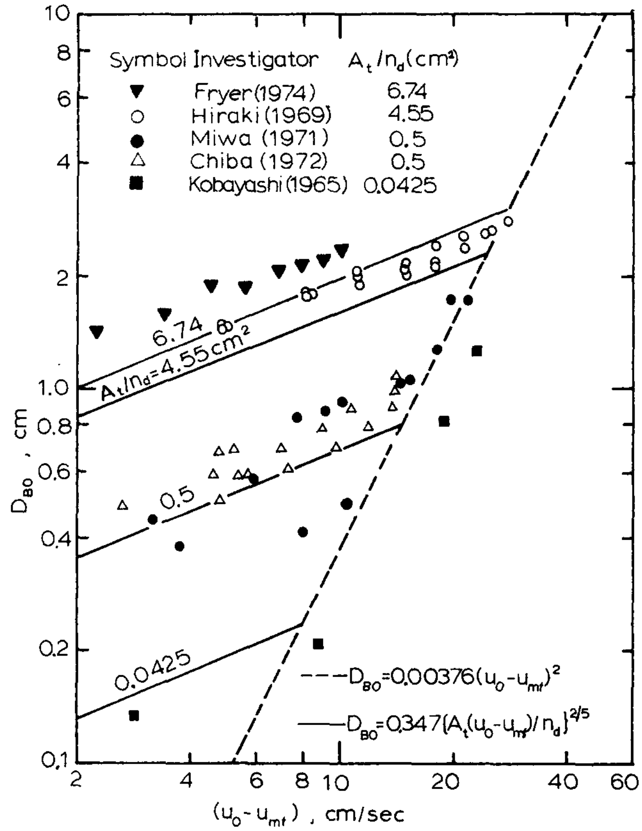

Fig. 2. Comparison of the observed initial bubble diameter with the diameter calculated by the correlations of Miwa et al. (1971).

THE MAXIMUM ATTAINABLE BUBBLE DIAMETER DUE TO THE TOTAL COALESCENCE OF BUBBLES, D_{BM}

Upon formation at distributor plate of the fluidized bed, the bubbles detach and are possibly swept toward the center line of the bed due to the presence of the bed wall. This picture of the bubble motion is supported among others by the data of Werther (1973) from a fluidized bed having porous plate distributors. As Werther's data in Figure 3 show the radial position r_p where the bubble flow rate reaches a maximum is a function of bed height. It can be seen from Figure 3 that the maximum flow rate occurs closer to the center line of the bed for greater heights above the distributor plate. Also as shown in the lower figure of Figure 3, the relationship between ln (r_p/R_t) and (h/D_t) is linear and nearly independent of the bed diameter. Thus the degree to which bubbles leaving the distributor are swept toward the center line of the bed seems to be a function only of the dimensionless height h/D_t.

As the bubbles in the fluidized bed are swept (or funneled) toward the center line of the bed, the bubbles will begin to grow by coalescence because of the increased bubble density at the center line. The ultimate limit of this process, if the bed height were tall enough, would be a single train of large bubbles rising along the center line of the bed. It is the diameter of these bubbles that is denoted by D_{BM}, the maximum attainable bubble diameter due to the total coalescence of bubbles.

The maximum attainable bubble size due to total coalescence of bubbles D_{BM} described here is in a sense a fictitious bubble diameter for a large diameter bed; this diameter can nevertheless be calculated. Consider bubbles of diameter D_{BM} traveling up the center line of the

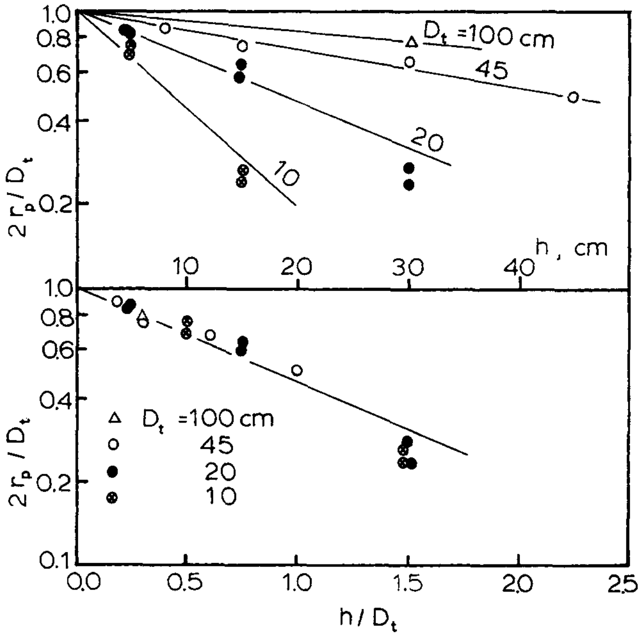

Fig. 3. Radial position in the fluidized bed where the maximum bubble flow rate occurs as a function of height above the distributor. The data are those of Werther (1973).

bed as described above. The distance between these bubbles l_{BM} is then given by

$$l_{BM} \equiv \alpha D_{BM} \tag{3}$$

where α is the number of bubble diameters which must be maintained between bubbles to prevent coalescence.

The velocity of a single train of bubbles rising along the center line, u_{BM}, is given by Davidson and Harrison (1963) and when $u_{BM} >> u_0 - u_{mf}$, can be simply written as

$$u_{BM} = 0.711\sqrt{g D_{BM}} + u_0 - u_{mf} \simeq 0.711\sqrt{g D_{BM}} \tag{4}$$

The maximum error for such an approximation is about 20% at the critical gas velocity corresponding to the bubbling condition of $\frac{u_0 - u_{mf}}{0.35\sqrt{g D_t}} = 0.2$.

The error becomes progressively smaller as $(u_0 - u_{mf})$ is decreased. Thus, making use of Equation (3) and (4), the frequency f_{BM} at which bubbles of diameter D_{BM} pass a fixed position is

$$f_{BM} = \frac{u_{BM}}{l_{BM}} = \frac{0.711}{\alpha}\sqrt{\frac{g}{D_{BM}}} \tag{5}$$

On the other hand, the superficial velocity of the bubble can be approximated by $(u_0 - u_{mf})$ and the frequency f_{BM} for bubbles of diameter D_{BM} rising in a single train along the center line of the bed can be shown as

$$V_{BM} f_{BM} = A_t(u_0 - u_{mf}) \quad \text{or} \quad f_{BM} = \frac{6A_t}{\pi D_{BM}^3}(u_0 - u_{mf}) \tag{6}$$

Eliminating f_{BM} from Equations (5) and (6) gives

$$D_{BM} = \left[\frac{6\alpha}{0.711\sqrt{g}\,\pi} A_t(u_0 - u_{mf})\right]^{2/5}$$
$$= 0.374\,(\alpha A_t(u_0 - u_{mf}))^{2/5} \tag{7}$$

Equation (7) gives D_{BM} in terms of the quantity α which can be estimated as follows: Leung (1972) has analyzed the data of Botterill et al. (1966) and observed that the ratio (D_{BM}/D_{B0}) is about 1.87 for bubbles injected into a fluidized bed through a single nozzle. For a single nozzle the initial bubble diameter D_{B0} may be given by Equation (1) as

$$D_{B0} = 0.347\,\{A_t(u_0 - u_{mf})\}^{2/5} \tag{8}$$

Hence

$$D_{BM} = (0.347)(1.87)\{A_t(u_0 - u_{mf})\}^{2/5} \tag{9}$$

Comparing Equation (9) with Equation (7) gives an estimate of α of 4.0.

A MODEL OF BUBBLE GROWTH IN FLUIDIZED BEDS

In the initial section of this paper, it was stated that the bubble diameter D_B depended upon four quantities D_{BM}, D_{B0}, D_t, and h. In this section the relationship between these variables will be shown explicitly.

Miwa et al. (1970) have suggested that the ratio $(D_{BM} - D_B)/(D_{BM} - D_{B0})$ varies exponentially with the height above the distributor. Also as was seen from Werther's data (1973) discussed above, the degree to which bubbles are swept toward the center line of the fluidized bed and hence the degree of coalescence is a function only of the dimensionless height h/D_t. Since the ratio $(D_{BM} - D_B)/(D_{BM} - D_{B0})$ can be considered as a measure of the degree of coalescence, the following equation for the growth of bubbles in fluidized beds is suggested:

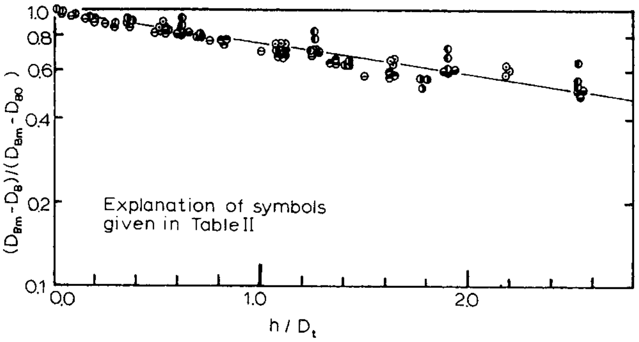

Fig. 4. Comparison of Equation (10) with experimental bubble size data showing that the quantity k in Equation (10) is practically constant.

$$\frac{D_{BM} - D_B}{D_{BM} - D_{B0}} = e^{-kh/D_t} \tag{10}$$

where k is a quantity which is to be determined.

The ratio $(D_{BM} - D_B)/(D_{BM} - D_{B0})$ in Equation (10) can be calculated using the experimentally observed bubble diameter D_B and estimating the initial bubble diameter D_{B0} by either Equation (1) or (2) as discussed in the previous section and the maximum bubble diameter attainable due to coalescence D_{BM} by Equation (7) using $\alpha = 4.0$.

Equation (10) suggests that plotting of experimental bubble diameter data as $\ln((D_{BM} - D_B)/(D_{BM} - D_{B0}))$ vs. h/D_t should yield a straight line if k is a constant. It can be seen in Figure 4 that the data of Rowe (1972) and Werther (1973), which cover a wide range of operating conditions, lie on a straight line when plotted in the manner suggested above; therefore, for all practical purposes the quantity k, which appears in Equation (10), will be assumed to be a constant.

The value of k in the proposed bubble growth equation [Equation (10)] was statistically determined from more than 400 data from various investigators. Table 2 lists the references for these data and summarizes the operating conditions under which the data were taken. The value of k which minimized the absolute value of the error between observed bubble size and the calculated diameter was found to be 0.30. For $k = 0.30$ the mean error and the standard deviation of Equation (10) from experimental data was 31% and 54%, respectively. Comparisons between the observed bubble diameter and the diameter calculated from Equation (10) (with $\alpha = 4.00$ and $k = 0.3$) are shown in Figures 5 and 6.

Some of the reasons for the scatter of data in Figures 5 and 6 are:

1. In most fluidized beds, bubbles usually have a wide size distribution at a given point in the bed (Miwa et al., 1971; Werther, 1973). It is common to observe bubble size ratios of two to three at a point of measurement in the bed. In addition, there is a considerable bubble size distribution in the radial direction of the bed (Kobsyoshi et al., 1965; Werther, 1973).

2. The small bubble size data obtained by capacitance probes in the bed are subject to considerable experimental error since small bubbles are comparable in dimensions to the probe.

3. Experiments carried out in small diameter beds are prone to inaccuracies due to slugging effects. Actually some of the data in Figure 6 were considered to be in the slug-flow or the transition region between free-bubbling and slug-flow. This reason explains partially the fact that the large diameter fluid-bed data presented in Figure 5 shows less scatter than the small bed data presented in

TABLE 2. SUMMARY OF EXPERIMENTAL CONDITIONS FOR THE BUBBLE DIAMETER IN THE FLUIDIZED BED

B: Bubble cap; T: tuyere; P_e: perforated plate; P_o: porous plate or screen

Investigators	Key	D_t, cm	Solid particles	d_p, cm	u_{mf}, cm/s	u_o/u_{mf}	Distributor, n_d
Werther (1973)	⊖	20	Quartz sand	0.0083	1.8	5	P_o
	⦶	100					
Chiba (1973)	△	20	Crushed silica	0.0089	0.53	10 ~ 39	P_e, 241
	◬			0.0210	2.85	2 ~ 8	
Geldart (1971)	○	30.8	Sand	0.0128	1.2	2.6 ~ 7.7	P_e, 3100
Rowe (1972)	◐	30 × 20*	Alumina	0.021	2.54	1.25 ~ 2.5	
	◑	30 × 30*	Carbon	0.0296	8.0	1.3 ~ 1.7	
	◒		Quartz	0.0135	2.75	2.2 ~ 6.6	P_o
	◓	30 × 20*	Ballotini	0.0325	8.0	1.6 ~ 2.4	
	⊙		Glass powder	0.0268	5.5	1.7 ~ 2.7	
Whitehead (1967)	◪	61 × 61*				1.8 ~ 6.9	4
	⧅	61 × 61*	Silica sand	0.015	2.5	2.8 ~ 6.6	T 16
	⊠	122 × 122*				3.2 ~ 6.2	64
	■	122 × 122*				2.1 ~ 6.3	16
Kunii (1967)	●	20	M.S. cat.	0.015	2.0	9.5	P_e, 79
	●	40				1.5 ~ 25	P_e, 314
Yasui (1958)	▲		Glass beads	0.0242	7.56	1.5 ~ 2.5	
	▲	10.2	Glass beads	0.0175	4.7	1.5 ~ 2.7	P_o
	▲		U.O.P. cat.	0.0060	0.418	2 ~ 10	
	▲		Coal	0.0450	19.4	1.5 ~ 1.75	
Toei (1965)	⊗	10 × 10*	Glass beads	0.0137	2.25	1.5 ~ 4.0	P_o
Kobayashi	▼	10.0	Crushed silica	0.0210	2.85	2 ~ 9.7	P_e, 1850
Miwa (1971)	▽	15.0	Sand	0.016	2.4	3.1 ~ 5.2	P_e, 37
Tomita (1971)	⟠	21.4					184
	◈	37.8	Sand	0.0202	4.0	4.25	P_e 575
	◆	59.9					1450
Baumgarten (1960)	◇	7.6	Glass beads	0.0074	0.727	2 ~ 84	P_o
Park (1969)	▼			0.0086	0.63	4 ~ 10	
	▼	10.0	Conductive coke	0.0156	1.83	1.5 ~ 6	
	▽			0.0344	6.8	1.5 ~ 3	P_o
Botton (1968)	⬭	50	Sand	0.0071	1.0**	2 ~ 15	P_e, 78
Fryer (1974)	⬯	22.9	Sand	0.0071	1.70	1.47	B 61

* Diameter of a cylinder having same cross-sectional area of the actual bed was used for calculation.
** Gas flow rate through the dense phase reported by Botton (1968).

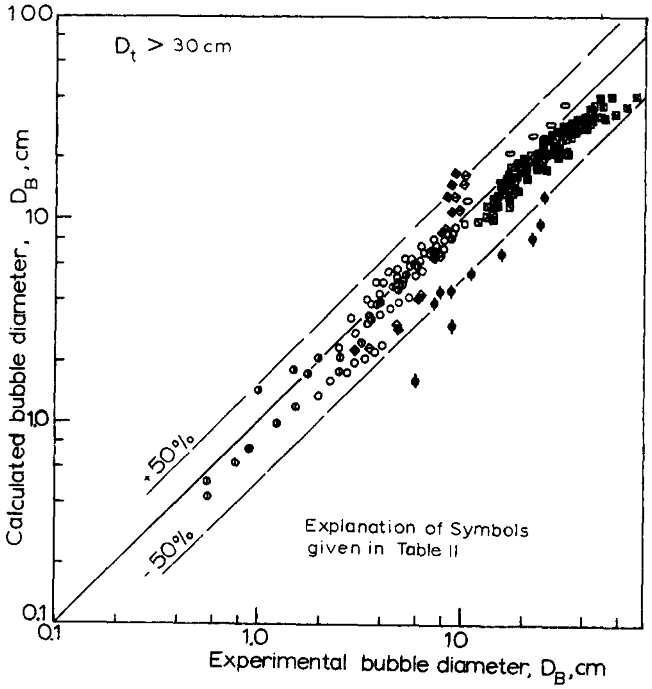

Fig. 5. Comparison of experimental bubble diameters observed in large diameter beds (D_t > 30 cm) with bubble diameters calculated from Equation (11).

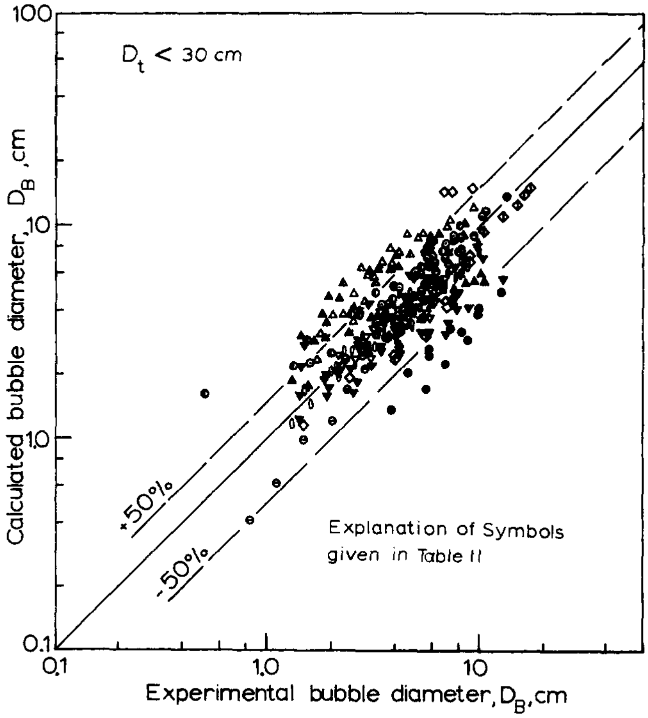

Fig. 6. Comparison of experimental bubble diameters observed in small diameter beds (D_t < 30 cm) with bubble diameters calculated from Equation (11).

Figure 6.

The final form of the bubble growth correlation is given by

$$(D_{BM} - D_B)/(D_{BM} - D_{B0}) = \exp[-0.3h/D_t] \quad (11)$$

The ranges of data from which this correlation was obtained are

$$0.5 \leq u_{mf} \leq 20 \text{ cm/s}$$
$$0.006 \leq d_p \leq 0.045 \text{ cm}$$
$$u_0 - u_{mf} \leq 48 \text{ cm/s}$$
$$D_t \leq 130 \text{ cm}$$

This correlation is good only up to $D_t = 130$ cm since all the data examined are from bed diameter less than 1.3 m. However, many industrial reactors and pilot-plant fluidized-bed reactors have the bed diameter in the same order of magnitude. This correlation relates, for the first time, the bubble diameter with bed diameter and is therefore believed useful in analysis and design of a large-scale unit.

DISCUSSION OF BUBBLE GROWTH CORRELATION

The original bubble growth correlation used by Wen and Kato (1969) on their Bubble Assemblage model was based on the study of Kobayashi et al. (1965). The correlation used is

$$D_B = 1.4\, d_p\, \rho_p\, u_0\, h/u_{mf} + D_{B0} \quad (12)$$

where D_{B0} is the initial bubble diameter just above the distributor and can be estimated by Equations (1) and (2).

Although there were some questions in regard to the effect of bed height as represented by Equation (12), this equation was used to calculate bubble diameters due to lack of accurate data and better correlation available at that time. Therefore, it would be of interest to examine how much the new correlation shown by Equation (11) improves over the original correlation of Equation (12). In Figure 7, the same set of data which appeared in Figure 5 for large diameter beds ($D_t > 30$ cm) are compared with those calculated using Equation (12). From a comparison of Figures 5 and 7, it is evident that the new correlation provides a substantial improvement over the original correlation, which tends to estimate too small bubble diameters at lower elevations and too large bubble diameters at higher elevations.

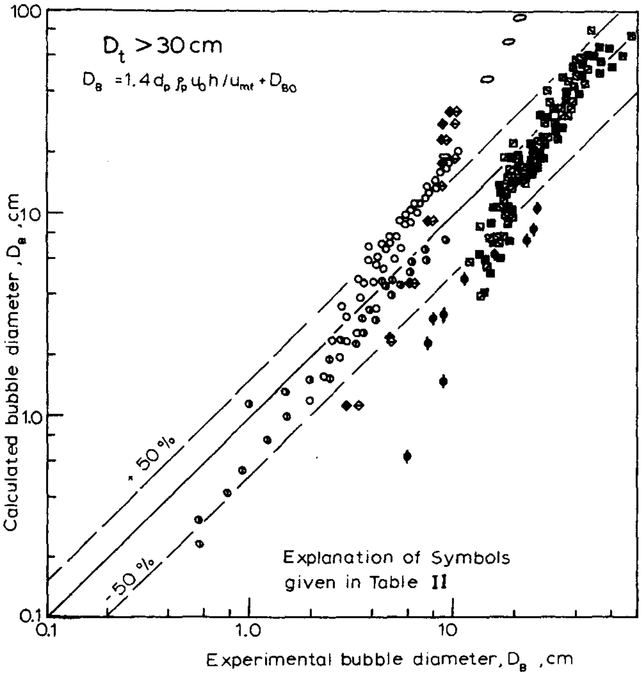

Fig. 7. Comparison of experimental bubble diameters observed in large diameter beds ($D_t > 30$ cm) with bubble diameters calculated from Equation (12).

There are several circumstances under which the proposed bubble growth correlation given by Equation (11) may not apply. One such circumstance is when the diameter of the bubble D_B exceeds 0.3 D_t. Under this condition ($D_B > 0.3\ D_t$) the bed may no longer be a freely bubbling bed. Hovmand and Davidson (1971) have presented data which indicate that for $0.2 \leq D_B/D_t \leq 0.5$ the bubbling in the fluidized bed may be in the transition region between the freely bubbling and the slug-flow regimes. Thus for $D_{BM} > 0.3\ D_t$, where D_{BM} is calculated from Equation (7), the proposed bubble growth correlation [Equation (11)] can be used to calculate the bubble diameter D_B until $D_B = 0.3\ D_t$. Beyond this point (that is, $D_B > 0.3\ D_t$), correlations for the slug-flow regime should probably be used.

As discussed previously, Equation (7) is developed based on the assumption that a single bubble track can be formed for fluidized beds with very large height-to-bed diameter ratio. This assumption seems applicable as long as $D_t < 130$ cm. However, when $D_t > 130$ cm, the concept of the maximum attainable bubble diameter D_{BM} may have to be modified although D_{BM} can still be treated as an empirical parameter. Whether the assumption made in developing Equation (7) can still hold or not for $D_t > 130$ cm remains to be validated in the future research.

Another situation where the proposed bubble growth correlation [Equation (11)] may not apply is when the diameter of the bubble D_B reaches the maximum bubble size determined by the stability of the bubble D_{BS} before the maximum attainable bubble diameter due to coalescence D_{BM} is reached.

Harrison et al. (1961) postulated that the maximum stable bubble diameter could be given by

$$D_{BS} = \left(\frac{U_T}{0.71}\right)^2 \cdot \frac{1}{g} \quad (13)$$

However, bubbles larger than that calculated from Equation (13) have been observed by Matsen (1973), Morooka et al. (1971), and Whitehead and Young (1967). This discrepancy is probably due in part to errors in estimating the effective terminal velocity U_T of very fine particles. However, until methods are developed further, only very crude estimates of the maximum stable bubble size D_{BS} can be made.

In the situation where $D_{BS} < D_{BM}$ (where D_{BM} is calculated from Equation (7)) the maximum attainable bubble diameter due to coalescence D_{BM} is a fictitious bubble size. However, it would seem reasonable to assume that the bubble growth for a bubble with a diameter D_B less than the maximum stable bubble diameter can be obtained by Equation (11). When the bubble diameter reaches the maximum stable bubble diameter it will tend to break-up and coalesce with the bubble diameter oscillating about the maximum stable bubble diameter. However, in the absence of reliable estimates of the maximum stable bubble diameter, this type of bubble growth is yet to be verified.

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NOTATION

A_t = cross-sectional area of the fluidized bed, cm^2
D_B = equivalent spherical bubble diameter having the same volume as that of a bubble, cm
D_{B0} = initial bubble diameter at the distributor, cm
D_{BM} = maximum bubble diameter due to total coalescences of bubbles, cm
D_{BS} = maximum bubble diameter determined by bubble stability, cm
D_t = diameter of the fluidized bed, cm
d_p = diameter of the fluidized particles, cm
f_{BM} = frequency at which bubbles of diameter D_{BM} pass a fixed position, 1/s
G = volumetric gas flow rate through a nozzle, cm^3/s
g = gravitational acceleration, cm/s^2
h = elevation above the distributor, cm
l_{BM} = minimum distance between bubble of diameter D_{BM} which is necessary to prevent coalescence, cm
n_d = number of orifice openings in the distributor
R_t = radius of the fluidized bed, cm
r_p = radial position, cm
u_0 = superficial gas velocity, cm/s
u_{BM} = bubble rising velocity for a bubble of diameter D_{BM}, cm/s
u_{mf} = minimum fluidization velocity, cm/s
U_T = terminal velocity of the solid particles, cm/s
V_{BM} = volume of a maximum bubble, cm^3
α = ratio of the vertical distance between bubbles having a diameter of D_{BM} to the maximum bubble diameter due to coalescence, D_{BM}
ρ_p = density of the particle, g/cm^3

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